Gravitational wakes in Saturn's rings

H. Salo

Department of Astronomy, University of Oulu, 90570 Oulu, Finland

THE outer parts of Saturn's rings display a variety of local non-uniformities in their particle distributions. Azimuthal brightness variations are seen in the A-ring[1,2], and may be attributable to the gravitational aggregation of particles into linear wakes that trail the rotation of the ring[3–5]; Voyager's stellar occultation experiments revealed large differences, on length scales as small as 150 m, in the surface density of particles[6]. Theoretical arguments[7,8] suggest that local instabilities may occur in both the A- and B-rings, the instability criterion depending on the velocity dispersion of ring particles and the orbital velocity and mass density in the ring. These arguments, however, are derived from the purely gravitational dynamics of an idealized, infinitesimally thin ring, made of identical particles. Here I use numerical simulations, including both gravitational interactions and dissipative impacts between particles, to study realistic models of Saturn's rings. For the C-ring there is no instability, but for the B- and A-rings gravitational wakes form. In the A-ring these wakes are so strong that particles trapped in them form metre-sized aggregate particles, which themselves lead to further instability. These different behaviours are consistent with the observational evidence.

The dynamical evolution of the main components of Saturn's rings is governed by the frequent impacts among the macroscopic icy particles; the local equilibrium state follows from the balance between dissipative energy loss in partially inelastic collisions and viscous gain of energy from the systematic velocity field. Laboratory measurements[9] indicate that collisions are very inelastic, with coefficient of restitution $\varepsilon < 0.5$, for impact velocities of only a few millimetres per second. Accordingly, purely collisional simulations predict little velocity dispersion at equilibrium, and therefore predict that the vertical thickness of the rings should only be only few times the dominant particle size. It is instructive to apply gravitational stability criteria to such idealized systems. According to Toomre[10], a differentially rotating, infinitely flat system is stable against all axisymmetric perturbations, as long as the radial velocity dispersion exceeds $\sigma_r > \sigma_{cr} = 3.36G\Sigma/\kappa$, where Σ denotes the surface mass density and κ the epicyclic frequency, equal to angular frequency Ω in a keplerian velocity field. For a system of identical particles each of density ρ and radius r, the surface density is related to normal optical thickness τ by $\Sigma = 4\rho r\tau/3$. Collisional simulations[11,12] indicate that $\sigma_r \approx 3r\Omega$ or smaller for any τ, and the stability criterion can thus be written $\tau_{cr} = 0.4(10^8/a\ \mathrm{m})^3 (0.9/\rho\ \mathrm{g\ cm^{-3}})$, giving the value of τ above which instability should occur at a distance a from the centre of Saturn. For the C-, B- and A-rings, $\tau_{cr} \approx 0.7$, 0.4 and 0.2, respectively, suggesting that although the C-ring, with mean $\tau \approx 0.15$, should be stable, both B- and A-rings ($\tau > 1.0$ and $\tau \approx 0.5$) might be susceptible to local instabilities.

This simple estimate for τ_{cr} should be considered with caution, as it ignores the increase of random velocities due to gravitational encounters[13–15]. In addition, recent collisional simulations with extended size distribution[16] indicate that σ_r can be as much as 5-fold for the smallest particles as compared with σ_r of the largest ones. This might stabilize at least their subpopulation, and possibly the whole system, by decreasing the effective surface density susceptible to perturbations. Any wakes formed would also scatter particles, and the net result might well be to alter the velocity dispersion in such a way that instabilities are eventually avoided[10].

Even if the axisymmetric Toomre stability criterion is fulfilled, systems with the ratio $Q_T = \sigma_r/\sigma_{cr}$ close to unity can still show a strong tendency to form transient density wakes. According to Julian and Toomre[17], these wakes appear in the form of trailing wavelets whose most prominent azimuthal wavelength is $\sim 2\lambda_{cr}$, where $\lambda_{cr} = 4\pi^2 G\Sigma/\kappa^2$ denotes the shortest axisymmetric wavelength stabilized by differential rotation alone. For Saturn's rings, $\lambda_{cr} \approx 70(a/10^8)^3(\Sigma/1{,}000)$, where a is in metres and Σ in kg m^{-2}, or about 6, 65 and 85 m for typical C-, B- and A-ring parameters. The expected pitch angle (angle between wavecrest and tangential direction) is ~10–15° for a keplerian velocity field, if the thickness of the system is negligible compared with λ_{cr}. At least for stellar systems, unless there is some external driving (for example in the form of an orbiting mass concentration), these wakes are rapidly damped[17]. Therefore, the formation of particle aggregates seems to be necessary to maintain persistent wakes. Formally, the classical Roche distance for ice particles $R_{Roche} = 136{,}500$ km, but Weidenshilling *et al.*[18] estimate that particle aggregates could form well inside R_{Roche}: in principle, a small particle can stick gravitationally to the surface of a slowly rotating large ice particle for $a >$ 70,000 km, or everywhere in the rings. As this estimate does not take into account the disrupting effects of collisions, however, the survival of particle aggregates is uncertain. On the other hand, external driving may not be necessary in the presence of dissipation: impacts cool the system continuously and thereby aid the amplification of new wakes (compare with stellar

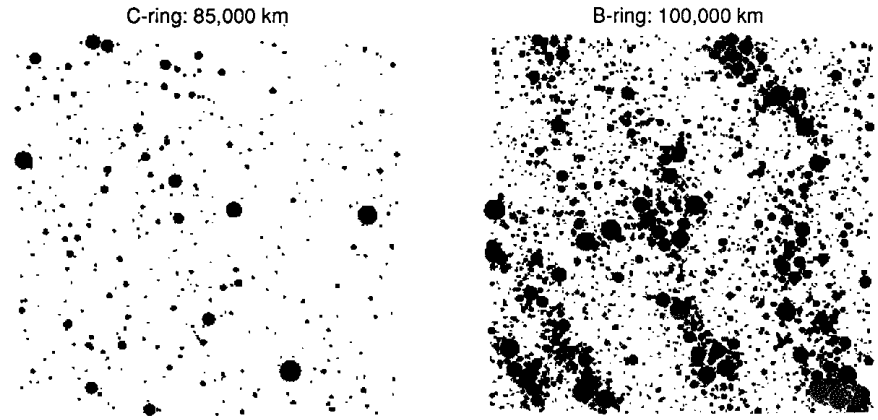

FIG. 1 Simulation for Saturn's C- and B-rings, for distances of $a = 85{,}000$ and 100,000 km from the centre of Saturn. I use the local simulation method[16], analogous to the stellar dynamical simulations of Toomre[20] and the planetary ring simulations of Wisdom and Tremaine[11]: all the calculations are restricted to a small co-moving cell inside the rings, with periodic boundary conditions taking into account the systematic shearing motion. Particle distributions after 10 orbital revolutions are displayed. The size distribution is approximated by a power-law with slope 3, extending over 50 cm $< r <$ 5 m, and each side of the (square) calculation cell is 170 m. Direction of the planet is to the left and mean orbital motion is upward. The gravitational force on each particle is calculated from all the other particles within 85 m. No softening is used, as the finite size of particles makes it unnecessary. Internal density of particles is taken to be $\rho = 0.9$ g cm^{-3}, and the surface densities are 120 and 960 kg m^{-2}, obtained by varying the total number of particles (400 and 3,200). For the coefficient of restitution, I assume a velocity-dependent elasticity model[9]: $\varepsilon(v_p) = (v_p/v_B)^{-0.234}$, $0.25 < \varepsilon < 1$, where v_p denotes the perpendicular component of impact velocity and $v_B = 0.01$ cm s^{-1}. Initially, random velocities correspond to equilibrium values in the non-gravitating case. The independence of the results from the cell size was checked by additional runs. The B-ring experiment required about 25 CPU hours on a IBM ES/9121-260 VF mainframe. Particle plots were produced by IDL software.